

# RankResurrection: From Rank-zero to Rank-Three Elliptic Curves Via Congruent Numbers

KOUAKOU Kouassi Vincent

*Applied Fundamental Sciences Department*

*Nangui ABROGOUA University.*

*Abidjan, Cote d'Ivoire*

*e-mail: kouakouassivincent@gmail.com*

02 BP 801 ABIBDJAN 02, Côte d'Ivoire

**Abstract.** We explore the construction of explicit families of high-rank elliptic curves connected to congruent numbers via polynomial interpolation. Building on foundational contributions from pioneers such as Mestre, Elkies, Lecacheux, and Dujella, we provide an algorithmic framework generating congruent numbers whose associated elliptic curves  $E_N$  admit rank  $\geq 3$ . Starting from  $E_p(\text{rank-0})$ , we identify explicit congruent numbers

$$N(m, n, p, q) = mnpq/2$$

yielding elliptic curves with rank exactly 3. Leveraging polynomial interpolation, specifically Lagrange's method, we unify these discrete instances into a parametric family

$$E(t) : y^2 = x^3 - N(t)^2x$$

capturing the observed rank phenomena. Our approach combines deep arithmetic insights with effective computational techniques, demonstrating rich algebraic structures and setting a path for further explorations of ranks in elliptic curve families. This provides a simple method to generate a family of high-rank elliptic curves associated with congruent numbers.

**MSC (2020):** 11G05, 11G40, 11Y50

**Keywords:** Elliptic Curves, Rank, Congruent Numbers, Lagrange interpolation, Parametric Families.

## References

- [1] Tunnell, J. B., A classical Diophantine problem and modular forms of weight  $3/2$ , *Invent. Math.* 72 (1983), 323–334.
- [2] N. Koblitz, *Introduction to Elliptic Curves and Modular Forms*, Graduate Texts in Mathematics, Vol. 97, Springer-Verlag, 1993.
- [3] P. Khanra, *The Congruent Number Problem and its Connection with Elliptic Curves*, Cambridge University Press, 2024.
- [4] Mestre, J.-F., Courbes elliptiques de rang  $\geq 12$  sur  $\mathbb{Q}(t)$ , *J. Number Theory* 53 (1995), 237–266.

- [5] J. Coates and A. Wiles, On the Conjecture of Birch and Swinnerton-Dyer. *Inventiones mathematicae*, 1987.
- [6] J. Silverman, *The Arithmetic of Elliptic Curves*, Springer, GTM 106, 2009.
- [7] J. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*, Springer Science and Business Media, 2013.
- [8] K. Conrad, Notes on Congruent Numbers, <https://kconrad.math.uconn.edu/blurbs/ugradnumthy/congnumber.pdf>
- [9] KOUAKOU Kouassi Vincent and Soro Kolo Fousséni, Two parametric families of congruent numbers from elliptic curves with quadratic point imposition, *JP Journal of Algebra, Number Theory and Applications* 64(6) (2025), 777-790. <https://doi.org/10.17654/0972555525043>
- [10] Kouakou Kouassi Vincent. (2025). Explicit Construction of a Parametric Family of Elliptic Curves of Rank 4 via a Quadratic Extension. *American Journal of Applied Mathematics*, 13(5), 344-347. <https://doi.org/10.11648/j.a.jam.20251305.14>
- [11] J. Lagrange, Construction d'une table de nombres congruents, *Mémoires de la S. M. F.*, tome 49-50, 125130, 1977.
- [12] KOUAKOU K. V., From Rank-Zero to Positive Rank Elliptic Curves Via The Unit Circle (Submitted)
- [13] SageMath 9.3, Available online at: <https://www.sagemath.org>